87. (a) \(f\) is continuous on \([-2, 4]\); the maximum value is 13, which occurs at \(x = 4\), and the minimum value is \(-3\), which occurs at \(x = 0\).
(b) \(f\) is continuous on \([1, 5]\); the maximum value is 1, which occurs at \(x = 1\), and the minimum value is 0.2, which occurs at \(x = 5\).
(c) \(f\) is continuous on \([-4, 1]\); the maximum value is 5, which occurs at \(x = -4\), and the minimum value is 2, which occurs at \(x = -1\).
(d) \(f\) is continuous on \([-4, 4]\); the maximum value is 5, which occurs at \(x = -4\), and the minimum value is 3, which occurs at \(x = 0\).

Section 1.3 Twelve Basic Functions

Exploration 1
1. The graphs of \(f(x) = \frac{1}{x}\) and \(f(x) = \ln x\) have vertical asymptotes at \(x = 0\).
2. The graph of \(g(x) = \frac{1}{x} + \ln x\) (shown below) does have a vertical asymptote at \(x = 0\).

3. The graphs of \(f(x) = \frac{1}{x}\), \(f(x) = e^x\), and \(f(x) = \frac{1}{1 + e^{-x}}\) have horizontal asymptotes at \(y = 0\).
4. The graph of \(g(x) = \frac{1}{x} + e^x\) (shown below) does have a horizontal asymptote at \(y = 0\).

5. Both \(f(x) = \frac{1}{x}\) and \(g(x) = \frac{1}{2x^2} - x = \frac{1}{x(x - 1)}\) have vertical asymptotes at \(x = 0\), but \(h(x) = f(x) + g(x)\) does not; it has a removable discontinuity.

Quick Review 1.3
1. \(59.34\)
2. \(5 - \pi\)
3. \(7 - \pi\)
4. \(3\)
5. \(0\)
6. \(1\)
7. \(3\)
8. \(-15\)
9. \(-4\)
10. \(\left|1 - \pi\right| = \pi - 1 - \pi = \pi - 1 = -1\)

Section 1.3 Exercises
1. \(y = x^3 + 1\); (e)
2. \(y = |x| - 2\); (g)
3. \(y = -\sqrt{x}\); (j)
4. \(y = -\sin x\) or \(y = \sin(-x)\); (a)
5. \(y = -x\); (i)
6. \(y = (x - 1)^2\); (f)
7. \(y = \text{int}(x + 1)\); (k)
8. \(y = -\frac{1}{x}\); (h)
9. \(y = (x + 2)^3\); (d)
10. \(y = e^x - 2\); (c)
11. \(2 - \frac{4}{1 + e^{-x}}\); (l)
12. \(y = \cos x + 1\); (b)
13. Exercise 8
14. Exercise 3
15. Exercises 7, 8
16. Exercise 7 (Remember that a continuous function is one that is continuous at every point in its domain.)
17. Exercises 2, 4, 6, 10, 11, 12
18. Exercises 3, 4, 11, 12
19. \(y = x, y = x^3, y = \frac{1}{x}, y = \sin x\)
20. \(y = x, y = x^3, y = \sqrt{x}, y = e^x, y = \ln x, y = \frac{1}{1 + e^{-x}}\)
21. \(y = x^2, y = \frac{1}{x}, y = |x|\)
22. \(y = \sin x, y = \cos x, y = \text{int}(x)\)
23. \(y = \frac{1}{x}, y = e^x, y = \frac{1}{1 + e^{-x}}\)
24. \(y = x, y = x^3, y = \ln x\)
25. \(y = \frac{1}{x}, y = \sin x, y = \cos x, y = \frac{1}{1 + e^{-x}}\)
26. \(y = x, y = x^3, y = \text{int}(x)\)
27. \(y = x, y = x^3, y = \frac{1}{x}, y = \sin x\)
28. \(y = \sin x, y = \cos x\)
29. Domain: All reals
   Range: \([-5, \infty)\)
   \([-10, 10]\) by \([-10, 10]\)

30. Domain: All reals
   Range: \([0, \infty)\)
   \([-10, 10]\) by \([-10, 10]\)

31. Domain: \((-6, \infty)\)
   Range: All reals
   \([-10, 10]\) by \([-10, 10]\)

32. Domain: \((-\infty, 0) \cup (0, \infty)\)
   Range: \((-\infty, 3) \cup (3, \infty)\)
   \([-5, 5]\) by \([-2, 8]\)

33. Domain: All reals
   Range: All integers
   \([-10, 10]\) by \([-10, 10]\)

34. Domain: All reals
   Range: \([0, \infty)\)
   \([-10, 10]\) by \([-10, 10]\)

35. \([0, 20]\) by \([-5, 5]\)
   (a) \(r(x)\) is increasing on \([10, \infty)\).
   (b) \(r(x)\) is neither odd nor even.
   (c) The one extreme is a minimum value of 0 at \(x = 10\).
   (d) \(r(x) = \sqrt{x} - 10\) is the square root function, shifted 10 units right.

36. \([0, 7]\) by \([2, 7]\)
   (a) \(f(x)\) is increasing on \([\frac{(2k - 1)\pi}{2}, \frac{(2k + 1)\pi}{2}]\) and decreasing on \([\frac{(2k + 1)\pi}{2}, \frac{(2k + 3)\pi}{2}]\) where \(k\) is an even integer.
   (b) \(f(x)\) is neither odd nor even.

(c) There are minimum values of 4 at \(x = \frac{(2k - 1)\pi}{2}\) and maximum values of 6 at \(x = \frac{(2k + 1)\pi}{2}\), where \(k\) is an even integer.

(d) \(f(x) = \sin(x) + 5\) is the sine function, \(\sin x\), shifted 5 units up.

37. \([-5, 5]\) by \([-1, 4]\)
   (a) \(f(x)\) is increasing on \((-\infty, \infty)\).
   (b) \(f(x)\) is neither odd nor even.
   (c) There are no extrema.
   (d) \(f(x) = \frac{3}{1 + e^{-x}}\) is the logistic function, \(\frac{1}{1 + e^{-x}}\), stretched vertically by a factor of 3.

38. \([-11.4, 7.4]\) by \([-2.2, 10.2]\)
   (a) \(q(x)\) is increasing on \((-\infty, \infty)\).
   (b) \(q(x)\) is neither odd nor even.
   (c) There are no extrema.
   (d) \(q(x) = e^x + 2\) is the exponential function, \(e^x\), shifted 2 units up.

39. \([-15, 15]\) by \([-20, 10]\)
   (a) \(h(x)\) is increasing on \([0, \infty)\) and decreasing on \((-\infty, 0]\).
   (b) \(h(x)\) is even, because it is symmetric about the y-axis.
   (c) The one extremum is a minimum value of 10 at \(x = 0\).
   (d) \(h(x) = \sqrt{|x|} - 10\) is the absolute value function, \(|x|\), shifted 10 units down.

40. \([0, 7]\) by \([-5, 5]\)
   (a) \(g(x)\) is increasing on \([\frac{(2k - 1)\pi}{2}, \frac{(2k + 1)\pi}{2}]\) and decreasing on \([\frac{(2k + 1)\pi}{2}, \frac{(2k + 3)\pi}{2}]\) where \(k\) is an integer.
Chapter 1 Functions and Graphs

44. \( g(x) \) is even, because it is symmetric about the y-axis.

(b) \( g(x) \) is even, because it is symmetric about the y-axis.

(c) There are minimum values of \(-4\) at \( x = (2k - 1)\pi \)
and maximum values of \( 4 \) at \( x = 2k\pi \), where \( k \) is an integer.

(d) \( g(x) = 4 \cos(x) \) is the cosine function, \( \cos x \),
stretched vertically by a factor of 4.

45. There are no points of discontinuity.

There are no points of discontinuity.

There are no points of discontinuity.

There are no points of discontinuity.

46. There is a point of discontinuity at \( x = 0 \).

47. There are no points of discontinuity.

48. There is a point of discontinuity at \( x = 0 \).

49. There are no points of discontinuity.

50. There are no points of discontinuity.
51. There is a point of discontinuity at $x=0$.

52. There are points of discontinuity at $x=2, 3, 4, 5, \ldots$.

53. (a) This is $g(x) = |x|$.  
(b) Squaring $x$ and taking the (positive) square root has the same effect as the absolute value function.

54. (a) This appears to be $f(x) = |x|$.  
(b) For example, $g(1) = 0.99 \neq f(1) = 1$.

55. (a) This is the function $f(x) = x$.  
(b) The fact that $\ln e^x = x$ shows that the natural logarithm function takes on arbitrarily large values. In particular, it takes on the value $L$ when $x = e^L$.

56. (a) 

(b) One possible answer: It is similar because it has discontinuities spaced at regular intervals. It is different because its domain is the set of positive real numbers, and because it is constant on intervals of the form $(k, k+1]$ instead of $[k, k+1)$, where $k$ is an integer.

57. The Greatest Integer Function $f(x) = \text{int}(x)$

<table>
<thead>
<tr>
<th>Weight (oz)</th>
<th>Cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>21</td>
<td>0.60</td>
</tr>
<tr>
<td>1.29</td>
<td>0.37</td>
</tr>
<tr>
<td>1.06</td>
<td></td>
</tr>
<tr>
<td>0.83</td>
<td></td>
</tr>
</tbody>
</table>

58. False. Because the greatest integer function is not one-to-one, its inverse relation is not a function.

59. True. The asymptotes are $x = 0$ and $x = 1$.

60. Because $3 - \frac{1}{x} \neq 3, 0 < \frac{5}{1 + e^{-x}} < 5, -4 \leq 4 \cos x \leq 4,$ and $\text{int}(x - 2)$ takes only integer values. The answer is A.

61. $3 < 3 + \frac{1}{1 + e^{-x}} < 4$. The answer is D.

62. By comparison of the graphs, the answer is C.

63. The answer is E. The others all have either a restricted domain or intervals where the function is decreasing or constant.
64. (a) Answers will vary.
   (b) In this window, it appears that $\sqrt{x} < x < x^2$.

65. (a) A product of two odd functions is even.
   (b) A product of two even functions is even.
   (c) A product of an odd function and an even function is odd.

66. Answers vary.

67. (a) Pepperoni count ought to be proportional to the area of the pizza, which is proportional to the square of the radius.
   (b) $12 = k(4)^2$ 
      $k = \frac{12}{4} = 3$ 
      $= 0.75$
   (c) Yes, very well.
   (d) The fact that the pepperoni count fits the expected quadratic model so perfectly suggests that the pizzeria uses such a chart. If repeated observations produced the same results, there would be little doubt.

68. (a) $y = e^x$ and $y = \ln x$
   (b) $y = x$ and $y = \frac{1}{x}$
   (c) With domain $[0, \infty)$, $y = x^2$ becomes the inverse of $y = \sqrt{x}$.

69. (a) At $x = 0, \frac{1}{x}$ does not exist, $e^x = 1, \ln x$ is not defined,

   $\cos x = 1$, and $\frac{1}{1 + e^{-x}} = 1$.

   (b) for $f(x) = x$, $f(x + y) = x + y = f(x) + f(y)$
   (c) for $f(x) = e^x$, $f(xy) = e^{xy} = e^{x}e^{y} = f(x) \cdot f(y)$
   (d) for $f(x) = \ln x, f(x + y) = \ln(xy) = \ln(x) + \ln(y) = f(x) + f(y)$
   (e) The odd functions: $x, x^3, \frac{1}{x}, \sin x$

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Section 1.4 Building Functions from Functions

Exploration 1

If $f = 2x - 3$ and $g = \frac{x + 3}{2}$, then

$f \circ g = 2\left(\frac{x + 3}{2}\right) - 3 = x + 3 - 3 = x$.

If $f = |2x + 4|$ and $g = \frac{(x - 2)(x + 2)}{2}$,

then $f \circ g = \frac{1}{2}\left|\frac{(x - 2)(x + 2)}{2} + 4\right|
= \left|\frac{x - 2}{2}\right| = \left|x^2 - 4 + 4\right| = \left|x^2\right| = x^2$.

If $f = \sqrt{x}$ and $g = x^2$, then $f \circ g = \sqrt{x^2} = 1$. Note, we use the absolute value of x because $g$ is defined for $-\infty < x < \infty$, while $f$ is defined only for positive values of x. The absolute value function is always positive. If $f = x^0$ and $g = x^{0.6}$, then $f \circ g = \left(x^{0.6}\right)^3 = x^3$.

If $f = x - 3$ and $g = \ln(e^x)$, then

$f \circ g = \ln(e^{x - 3}) = 3 - x = 3 \ln x - x = 3 + \ln x - 3 = \ln x$.

If $f = 2 \sin x$ and $g = 2$, then $f \circ g = 2 \sin 2x = \sin \left(2\left(\frac{x}{2}\right)\right) = \sin x$. This is the double angle formula (see Section 5.4). You can see this graphically.

If $f = 1 - 2x^2$ and $g = \sin \left(\frac{x}{2}\right)$, then

$f \circ g = 1 - 2\left(\sin^2 \left(\frac{x}{2}\right)\right) = \cos \left(\frac{x}{2}\right) = \cos x$.

(The double angle formula for $\cos x$ is $\cos 2x = \cos^2 x - \sin^2 x = (1 - \sin^2 x) - \sin^2 x = 1 - \sin^2 x$. See Section 5.3.)

This can be seen graphically:
Section 1.4 Building Functions from Functions

5. \[(f/g)(x) = \frac{\sqrt{x+3}}{x^2}; x+3 \geq 0 \text{ and } x \neq 0, \]
so the domain is \([-3, 0) \cup (0, \infty)\).

\[(g/f)(x) = \frac{x}{\sqrt{x+3}}; x+3 \geq 0, \]
so the domain is \((-\infty, 0) \cup (0, \infty)\).

6. \[(f/g)(x) = \frac{\sqrt{x-2}}{\sqrt{x+4}}; x-2 \geq 0 \text{ and } x+4 \geq 0, \]
so the domain is \([2, \infty)\).

\[(g/f)(x) = \frac{\sqrt{x+4}}{\sqrt{x-2}}; x+4 \geq 0 \text{ and } x-2 \geq 0, \]
so the domain is \((2, \infty)\).

7. \[(f/g)(x) = \frac{x^2}{\sqrt{1-x^2}}. \]
The denominator cannot be zero and the term under the square root must be positive, so \(1-x^2 > 0\). Therefore, \(x^2 < 1\), which means that \(-1 < x < 1\). The domain is \((-1, 1)\).

8. \[(f/g)(x) = \frac{x^3}{\sqrt{1-x^4}}. \]
The denominator cannot be zero, so \(x \neq 0\). Therefore, \(-1 < x < 1\). This means that \(x \neq 1\). There are no restrictions on \(x\) in the numerator. The domain is \((-\infty, 1) \cup (1, \infty)\).

9. \[(f/g)(x) = \frac{\sqrt{x+3}}{x^2}; x+3 \geq 0 \text{ and } x \neq 0, \]
\[(g/f)(x) = \frac{x}{\sqrt{x+3}}; x+3 \geq 0, \]
so the domain is \((-\infty, 0) \cup (0, \infty)\).

10. \[(f/g)(3) = f(g(3)) = f(9-3) = f(6) = 3; \]
\[(g/f)(-2) = g(f(-2)) = g(3) = 3 \]
\[(g*f)(2) = g(f(2)) = g(4) = 4. \]

Quick Review 1.4

1. \((-\infty, -3) \cup (-3, \infty)\)
2. \((1, \infty)\)
3. \((-\infty, 5]\)
4. \((1/2, \infty)\)
5. \([1, \infty)\)
6. \([-1, 1)\)
7. \((-\infty, \infty)\)
8. \((-\infty, 0) \cup (0, \infty)\)
9. \((-1, 1)\)
10. \((-\infty, \infty)\)

Section 1.4 Exercises

1. \((f + g)(x) = 2x - 1 + x^2; (f - g)(x) = 2x - 1 - x^2; \)
\((f/g)(x) = (2x - 1)(x^3) = 2x^4 - x^3. \)
There are no restrictions on any of the domains, so all three domains are \((-\infty, \infty)\).

2. \((f + g)(x) = (x - 1)^2 + 3 - x = \)
\(x^2 - 2x + 1 + 3 - x = x^2 - 3x + 4; \)
\((f - g)(x) = (x - 1)^2 - 3 + x = \)
\(x^2 - 2x + 1 - 3 + x = x^2 - x - 2; \)
\((f/g)(x) = (x - 1)^2(3 - x) = (x^2 - 2x + 1)(3 - x) = 3x^2 - 3x^3 + 6x + 2x^2 + 3 - x = -x^3 + 5x^2 - 7x + 3. \)
There are no restrictions on any of the domains, so all three domains are \((-\infty, \infty)\).

3. \((f + g)(x) = \sqrt{x} + \sin x; (f - g)(x) = \sqrt{x} - \sin x; \)
\((f/g)(x) = \sqrt{x} \sin x. \)
Domain in each case is \([0, \infty)\). For \(\sqrt{x}, x \geq 0. \) For \(\sin x, -\infty < x < \infty. \)

4. \((f + g)(x) = \sqrt{x + 5} + |x + 3|; \)
\((f - g)(x) = \sqrt{x + 5} - |x + 3|; \)
\((f/g)(x) = \sqrt{x + 5}|x + 3|. \)
All three expressions contain \(\sqrt{x + 5}, \) so \(x + 5 \geq 0 \) and \(x \geq -5; \) all three domains are \([-5, \infty)\). For \(|x + 3|, -\infty < x < \infty. \)
15. $f(g(x)) = 3(x - 1) + 2 = 3x - 3 + 2 = 3x - 1$.
Because both $f$ and $g$ have domain $(-\infty, \infty)$, the domain of $f(g(x))$ is $(-\infty, \infty)$.
$g(f(x)) = (3x + 2) - 1 = 3x + 1$; again, the domain is $(-\infty, \infty)$.

16. $f(g(x)) = \left(\frac{1}{x - 1}\right)^2 - 1 = \frac{1}{(x - 1)^2} - 1$. The domain of $g$ is $x \neq 1$, while the domain of $f$ is $(-\infty, \infty)$, so the domain of $f(g(x))$ is $x \neq 1$, or $(-\infty, 1) \cup (1, \infty)$.
$g(f(x)) = \frac{1}{(x^2 - 1) - 1} = \frac{1}{x^2 - 2}$. The domain of $f$ is $(-\infty, 1) \cup (1, \infty)$, while the domain of $g$ is $(-\infty, \infty)$, so the domain of $g(f(x))$ requires that $f(x) = 1$. This means $x^2 - 2 = 1$, or $x^2 = 3$, which means $x = -\sqrt{3}$ or $x = \sqrt{3}$. Therefore the domain of $g(f(x))$ is $(-\infty, \sqrt{3}) \cup (\sqrt{3}, \infty)$.

17. $f(g(x)) = (\sqrt{x + 1})^2 - 2 = x + 1 - 2 = x - 1$. The domain of $g$ is $x \geq -1$, while the domain of $f$ is $(-\infty, \infty)$, so the domain of $f(g(x))$ is $x \geq -1$, or $[-1, \infty)$.
$g(f(x)) = \sqrt{x^2 - 2} + 1 = \sqrt{x^2 - 1}$. The domain of $f$ is $(-\infty, \infty)$, while the domain of $g$ is $[-1, \infty)$, so $g(f(x))$ requires that $f(x) = -1$. This means $x^2 - 2 = -1$, or $x^2 = 1$, which means $x = -1$ or $x = 1$. Therefore the domain of $g(f(x))$ is $(-1, -1) \cup (1, \infty)$.

18. $f(g(x)) = \frac{1}{\sqrt{x - 1} - 1}$. The domain of $g$ is $x \geq 0$, while the domain of $f$ is $(-\infty, 1) \cup (1, \infty)$, so $f(g(x))$ requires that $x \geq 0$ and $g(x) \neq 1$, or $x \neq 0$ and $x \neq 1$. The domain of $f(g(x))$ is $[0, 1) \cup (1, \infty)$.
$g(f(x)) = \frac{1}{\sqrt{x - 1} - 1}$. The domain of $g$ is $x \neq 1$, while the domain of $f$ is $[0, \infty)$, so $g(f(x))$ requires that $x \neq 1$ and $f(x) \geq 0$, or $f(x) = 1$ and $\frac{1}{x - 1} \geq 0$. The latter occurs if $x - 1 \geq 0$, so the domain of $g(f(x))$ is $(1, \infty)$.

19. $f(g(x)) = (\sqrt{1 - x^3})^2 = 1 - x^3$; the domain is $[-1, 1]$.
$g(f(x)) = \sqrt{1 - x^3} - x^3 = \sqrt{1 - x^3}$; the domain is $[-1, 1]$.

20. $f(g(x)) = (\sqrt{1 - x^3})^2 = 1 - x^3$; the domain is $(-\infty, \infty)$.
$g(f(x)) = \sqrt{1 - x^3} - x^3 = \sqrt{1 - x^3}$; the domain is $(-\infty, \infty)$.

21. $f(g(x)) = \frac{1}{3x} = \frac{1}{2(1/3x)} = \frac{3}{2x}$; the domain is $(0, \infty)$. \[ g(f(x)) = \frac{1}{2x} \]
the domain is $(-\infty, 0) \cup (0, \infty)$.

22. $f(g(x)) = \frac{1}{x - 1} = \frac{1}{1/(x - 1)} + 1 = \frac{1}{(1/(x - 1))/x - 1} = \frac{x - 1}{x}$. The domain is all reals except 0 and 1.

\[ g(f(x)) = g\left(\frac{1}{x + 1}\right) = \frac{1}{(1/(x + 1)) - 1} = \frac{1}{1/(x + 1) - (x + 1)} = \frac{x + 1}{x(x + 1)} = \frac{1}{x}; \]
the domain is all reals except 0 and 1.

23. One possibility: $f(x) = x^2$ and $g(x) = x^2 - x$.
24. One possibility: $f(x) = x^2$ and $g(x) = x^2$.
25. One possibility: $f(x) = |x|$ and $g(x) = 3x - 2$.
26. One possibility: $f(x) = x/3$ and $g(x) = x^3 - 5x + 3$.
27. One possibility: $f(x) = x^2 + 2$ and $g(x) = x^2 - x$.
28. One possibility: $f(x) = x^2$ and $g(x) = x - 3$.
29. One possibility: $f(x) = x^2 + 1$ and $g(x) = \tan x$.
30. One possibility: $f(x) = \cos x$ and $g(x) = \sqrt{x}$.

31. $r = 48 + 0.03t$ in., so $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(48 + 0.03t)^3$; when $t = 300$,
$V = \frac{4}{3}\pi(48 + 9) = 246,924\pi \approx 775,734.6$ in.$^3$.

32. The original diameter of each snowball is 4 in, so the original radius is 2 in and the original volume $V = \frac{4}{3}\pi r^3 \approx 33.5$ in.$^3$. The new volume is $V = 33.5 - t$, where $r$ is the number of 40-day periods. At the end of 360 days, the new volume is $V = 33.5 - 9 = 24.5$. Since $V = \frac{4}{3}\pi r^3$, we know that $r = \sqrt[3]{\frac{3V}{4\pi}} \approx 1.8$ in.

The diameter, then, is 2 times $r$, or $\approx 3.6$ in.

33. The initial area is $(5)(7) = 35$ km$^2$. The new length and width are $l = 5 + 2t$ and $w = 7 + 2t$, so $A = lw = (5 + 2t)(7 + 2t)$. Solve $(7 + 2t)(5 + 2t) = 175$ (5 times its original size), either graphically or algebraically; the positive solution is $t \approx 3.63$ sec.

34. The initial volume is $(5)(7)(3) = 105$ cm$^3$. The new length, width, and height are $l = 5 + 2t$, $w = 7 + 2t$, and $h = 3 + 2t$, so the new volume is $V = (5 + 2t)(7 + 2t)(3 + 2t)$. Solve graphically $(5 + 2t)(7 + 2t)(3 + 2t) = 525$ (5 times the original volume); $t \approx 1.62$ sec.

35. $3(1) + 4(1) = 3 + 4 = 7 \neq 5$
$3(4) + 4(-2) = 12 - 8 = 4 \neq 5$
$3(3) + 4(-1) = 9 - 4 = 5$
The answer is $(3, -1)$.

36. $(5)^2 + (1)^2 = 25 + 1 = 26 \neq 25$
$(3)^2 + (4)^2 = 9 + 16 = 25$
$(0)^2 + (-5)^2 = 0 + 25 = 25$
The answer is $(3, 4)$ and $(0, -5)$.

37. $x^2 = 25 - x^2$, $y = -\sqrt{25 - x^2}$ and $y = -\sqrt{25 - x^2}$

38. $x^2 = 25 - x$, $y = \sqrt{25 - x}$ and $y = -\sqrt{25 - x}$

39. $x^2 = x^2 - 25$, $y = \sqrt{x^2 - 25}$ and $y = -\sqrt{x^2 - 25}$

40. $x^2 = 3x^2 - 25$, $y = \sqrt{3x^2 - 25}$ and $y = -\sqrt{3x^2 - 25}$

41. $x + |y| = 1 \Rightarrow |y| = -x + 1 \Rightarrow y = -x + 1$ or $y = -(x - 1)$, $y = 1 - x$ and $y = x - 1$

42. $x - |y| = 1 \Rightarrow |y| = x - 1 \Rightarrow y = x - 1$ or $y = -(x - 1)$, $y = x - 1$ and $y = 1 - x$
43. \( y^2 = x^2 \Rightarrow y = x \) and \( y = -x \) or \( y = |x| \) and \( y = -|x| \)

44. \( y^2 = x \Rightarrow y = \sqrt{x} \) and \( y = -\sqrt{x} \)

45. False. If \( g(x) = 0 \), then \( \frac{f}{g} \)(x) is not defined and 0 is not in the domain of \( \left( \frac{f}{g} \right)(x) \), even though 0 may be in the domains of both \( f(x) \) and \( g(x) \).

46. False. For a number to be in the domain of \( f(2x) \) and \( g(x) \), it must be in the domains of both \( f(x) \) and \( g(x) \), so that \( f(x) \) and \( g(x) \) are both defined.

47. Composition of functions isn’t necessarily commutative.

The answer is C.

48. \( g(x) = \sqrt{4 - x} \) cannot equal zero and the term under the square root must be positive, so \( x \) can be any real number less than 4. The answer is A.

49. \((f \circ f)(x) = f(x^2 + 1) = (x^2 + 1)^2 + 1 = (x^2 + 2x + 1) + 1 = x^2 + 2x + 2 \). The answer is E.

50. \( y = |x| \Rightarrow y = x, y = -x \); \( y = -x \Rightarrow x = y, y = -y \) or \( x = y \Rightarrow y^2 = x^2 \). The answer is B.

51. If \( f(x) = e^x \) and \( g(x) = 2 \ln x \), then \( f(g(x)) = f(2 \ln x) = e^{2 \ln x} = (e^{\ln 2})^x = x^2 \). The domain is \((0, \infty)\).

If \( f(x) = (x^2 + 2)^2 \) and \( g(x) = \sqrt{x - 2} \), then

\[
\frac{d}{dx} \left( (x^2 + 2)^2 \right) = \frac{d}{dx} \left( (x^2 + 2) \right)^2 = 2(x^2 + 2)(2x) = 4x(x^2 + 2).
\]

\[
\frac{d}{dx} \left( \sqrt{x - 2} \right) = \frac{1}{2\sqrt{x - 2}}.
\]

So

\[
\frac{d}{dx} \left( f(x) \right) \cdot \frac{d}{dx} \left( g(x) \right) = 4x(x^2 + 2) \cdot \frac{1}{2\sqrt{x - 2}} = 2x(x^2 + 2) \cdot \frac{1}{\sqrt{x - 2}}.
\]

52. (a) \((f \circ g)(x) = x^4 - 1 = (x^2 + 1)(x^2 - 1) = f(x) \cdot (x^2 - 1), \) so \( g(x) = x^2 - 1 \).

(b) \((f + g)(x) = 3x^2 - (x + 1) = 2x^2 - 1 = g(x) \).

(c) \((f/g)(x) = 1 \Rightarrow f(x) = g(x) \). So \( g(x) = x^2 + 1 \).

(d) \( f(g(x)) = 9x^4 + 1 \) and \( f(x) = x^2 + 1 \). If \( g(x) = 3x^2, \) then \( f(g(x)) = (3x^2)^2 = 9x^4 + 1 \).

(e) \( g(f(x)) = 9x^4 + 1 \) and \( f(x) = x^2 + 1 \). If \( g(x) = 9x^4 + 1 \), \( (x^2 + 1)^2 = (x^2 + 1)^2 = 9x^4 + 1 \).

53. (a) \((f + g)(x) = (g + f)(x) = f(x) \) if \( g(x) = 0 \).

(b) \((f/g)(x) = f(g(x)) = f(x) \) if \( g(x) = 1 \).

(c) \((f + g)(x) = (g + f)(x) = f(x) \) if \( g(x) = x \).

54. Yes, by definition, function composition is associative. That is, \((f \circ (g \circ h))(x) = f((g \circ h)(x)) = ((f \circ g) \circ h)(x) \).

55. \( y^2 + x^2y - 5 = 0 \). Using the quadratic formula,

\[
y = \frac{-x^2 \pm \sqrt{(x^2)^2 - 4(1)(-5)}}{2} = \frac{-x^2 \pm \sqrt{x^2 + 20}}{2}.
\]

56. \( y^2 + x^2y - 5 = 0 \). Using the quadratic formula,

\[
y = \frac{-x^2 \pm \sqrt{(x^2)^2 - 4(1)(-5)}}{2} = \frac{-x^2 \pm \sqrt{x^2 + 20}}{2}.
\]

**Section 1.5 Parametric Relations and Inverses**

**Exploration 1**

1. \( T \) starts at \(-4, \) at the point \((8, -3) \). It stops at \( T = 2, \) at the point \((8, 3) \). 61 points are computed.
Chapter 2 Polynomial, Power, and Rational Functions

(c) \( f(x) = \begin{cases} 1 & \text{if } x \neq -2, 0 \\ \text{undefined} & \text{if } x = -2 \text{ or } x = 0 \end{cases} \)

(d) The graph appears to be the horizontal line \( y = 1 \) with holes at \( x = -2 \) and \( x = 0 \).

This matches the definition in part (c).

52. \( y = 1 + \frac{1}{1 + x} \)
   \( y(1 + x) = 1 + x + 1 \)
   \( y + xy = x + 2 \)
   \( xy - x = 2 - y \)
   \( x = \frac{2 - y}{y - 1} \)

53. \( y = 1 - \frac{1}{1 - x} \)
   \( y(1 - x) = 1 - x - 1 \)
   \( y - xy = -x \)
   \( x = \frac{y - xy}{y - 1} \)

54. \( y = 1 + \frac{1}{1 + \frac{1}{x}} \)
   \( y(1 + x) = 1 + \frac{1}{1 + x} \)
   \( y + xy = 2x + 1 \)
   \( xy - 2x = 1 - y \)
   \( x = \frac{1 - y}{y - 2} \)

55. \( y = 1 + \frac{1}{1 + \frac{1}{x}} \)
   \( y = 1 + \frac{1}{1 - x + \frac{1}{x}} \)
   \( x = \frac{1 - x}{y - 2} \)

Section 2.8 Solving Inequalities in One Variable

Exploration 1
1. (a) \( (+)(-)(+)(-)(+)(+)(+) \)
   \( \text{Negative} \quad \text{Negative} \quad \text{Positive} \quad \text{Positive} \quad \text{Negative} \quad \text{Positive} \quad \text{Positive} \)

2. (a) \( (-)(+)(-)(-)(-)(+)(+) \)
   \( \text{Positive} \quad \text{Positive} \quad \text{Negative} \quad \text{Negative} \quad \text{Positive} \quad \text{Positive} \quad \text{Positive} \)

3. (a) \( (+)(+)(-)(+)(-)(+)(+)(+) \)
   \( \text{Positive} \quad \text{Positive} \quad \text{Negative} \quad \text{Negative} \quad \text{Positive} \quad \text{Positive} \quad \text{Positive} \quad \text{Positive} \)

Quick Review 2.8
1. \( \lim_{x \to \infty} f(x) = \infty, \quad \lim_{x \to -\infty} f(x) = -\infty \)
2. \( \lim_{x \to \infty} f(x) = -\infty, \quad \lim_{x \to -\infty} f(x) = -\infty \)
3. \( \lim_{x \to 0} g(x) = \infty, \quad \lim_{x \to \infty} g(x) = \infty \)
4. \( \lim_{x \to 0} g(x) = \infty, \quad \lim_{x \to \infty} g(x) = -\infty \)
5. \( \frac{x^3 + 5}{x} \)
6. \( x^3 - 3 \)

7. \( \frac{x(x - 3) - 2(2x + 1)}{(2x + 1)(x - 3)} = \frac{x^2 - 3x - 4x - 2}{(2x + 1)(x - 3)} = \frac{x^2 - 3x - 4x - 2}{2x^2 - 5x - 3} \)
8. \[
\frac{x(3x - 4) + (x + 1)(x - 1)}{(x - 1)(3x - 4)} = \frac{3x^2 - 4x + x^2 - 1}{(x - 1)(3x - 4)} = \frac{4x^2 - 4x - 1}{3x^2 - 7x + 4}
\]

9. (a) \(\pm 1, \pm 3\) or \(\pm 1, \pm \frac{1}{2}, \pm 3, \pm \frac{3}{2}\)

(b) A graph suggests that \(-1\) and \(\frac{3}{2}\) are good candidates for zeros.

(c) A graph suggests that \(-2\) and \(1\) are good candidates for zeros.

10. (a) \(\pm 1, \pm 3\) or \(\pm 1, \pm \frac{1}{2}, \pm \frac{2}{3}, \pm \frac{4}{3}, \pm \frac{8}{3}\)

Section 2.8 Solving Inequalities in One Variable

4. (a) \(f(x) = 0\) when \(x = \pm \frac{3}{5}\)

(b) \(f(x) > 0\) when \(x < \pm \frac{3}{5}\) or \(x > 1\)

(c) \(f(x) < 0\) when \(-\frac{3}{5} < x < 1\)

5. (a) \(f(x) = 0\) when \(x = \pm 8, \pm 1\)

(b) \(f(x) > 0\) when \(-1 < x < 8\) or \(x > 9\)

(c) \(f(x) < 0\) when \(x < -1\)

6. (a) \(f(x) = 0\) when \(x = \pm 2, \pm 9\)

(b) \(f(x) > 0\) when \(-2 < x < 9\) or \(x > 9\)

(c) \(f(x) < 0\) when \(x < -2\)

7. \((x + 1)(x - 3)^2 = 0\) when \(x = -1, 3\)

8. \((2x - 1)(x - 2)(3x - 4) = 0\) when \(x = -\frac{1}{2}, \frac{4}{3}\)

By the sign chart, the solution of \((x + 1)(x - 3)^2 > 0\) is \((-\infty, -1) \cup (3, \infty)\).

9. \((x + 1)(x - 3)^2 = 0\) when \(x = -1, 1, 2\)

By the sign chart, the solution of \((x + 1)(x - 1)(x - 2) = 0\) is \((-\infty, -1) \cup (1, 2)\).

10. \((2x - 7)(x^2 - 4x + 4) = (2x - 7)(x - 2)^2 = 0\) when \(x = 7\)

By the sign chart, the solution of \((2x - 7)(x - 2)^2 > 0\) is \(\left(-\frac{1}{2}, \frac{4}{3}\right) \cup \left[\frac{7}{2}, \infty\right)\).
11. By the Rational Zeros Theorem, the possible rational zeros are \( \pm 1, \pm \frac{1}{2}, \pm 2, \pm 3, \pm \frac{3}{2}, \pm 6 \). A graph suggests that \(-2, \frac{1}{2}, 3 \) are good candidates to be zeros.

\[
\begin{array}{c|cccc}
-2 & 2 & -3 & -11 & 6 \\
3 & 2 & -7 & 3 & 0 \\
\end{array}
\]

\( 2x^3 - 3x^2 - 11x + 6 = (x + 2)(x - 3)(2x - 1) = 0 \)

when \( x = -2, \frac{1}{2}, 3 \).

By the sign chart, the solution of \((x + 2)(x - 3)(2x - 1) = 0\) is \((-3, 3) \cup \left[ -\frac{1}{2}, 3 \right]\).

12. By the Rational Zeros Theorem, the possible rational zeros are \( \pm 1, \pm 2, \pm 3, \pm 6 \). A graph suggests that \(-1, 2, 3 \) are good candidates to be zeros.

\[
\begin{array}{c|cccc}
-1 & 1 & -4 & 1 & 6 \\
2 & 1 & -5 & 6 & 0 \\
\end{array}
\]

\( x^3 - 4x^2 + x + 6 = (x + 1)(x - 2)(x - 3) = 0 \)

when \( x = -1, 2, 3, 3 \).

By the sign chart, the solution of \((x + 1)(x - 2)(x - 3) = 0\) is \((-\infty, -1) \cup \{2, 3\}\).

13. The zeros of \( f(x) = x^3 - x^2 - 2x \) appear to be \(-1, 0, 2\). Substituting these values into \( f \) confirms this. The graph shows that the solution of \( x^3 - x^2 - 2x = 0 \) is \([-1, 0] \cup \{2, \infty\}\).

14. The zeros of \( f(x) = 2x^3 - 5x^2 + 3x \) appear to be \( 0, 1, \frac{3}{2} \). Substituting these values into \( f \) confirms this. The graph shows that the solution of \( 2x^3 - 5x^2 + 3x < 0 \) is \((-\infty, 0) \cup \left( 1, \frac{3}{2} \right)\).

15. The zeros of \( f(x) = 2x^3 - 5x^2 - x + 6 \) appear to be \(-1, \frac{3}{2}, 2\). Substituting these values into \( f \) confirms this. The graph shows that the solution of \( 2x^3 - 5x^2 - x + 6 > 0 \) is \([-1, \frac{3}{2}) \cup (2, \infty)\).

16. The zeros of \( f(x) = x^3 - 4x^2 - x + 4 \) appear to be \(-1, 1, 4\). Substituting these values into \( f \) confirms this. The graph shows that the solution of \( x^3 - 4x^2 - x + 4 \leq 0 \) is \((-\infty, -1] \cup [1, 4]\).

17. The only zero of \( f(x) = 3x^3 - 2x^2 - x + 6 \) is found graphically to be \( x \approx -1.15 \). The graph shows that the solution of \( 3x^3 - 2x^2 - x + 6 \geq 0 \) is approximately \([-1.15, \infty)\).

18. The only zero of \( f(x) = -x^3 - 3x^2 - 9x + 4 \) is found graphically to be \( x \approx 0.39 \). The graph shows that the solution of \(-x^3 - 3x^2 - 9x + 4 < 0 \) is approximately \((0.39, \infty)\).
19. The zeros of \( f(x) = 2x^4 - 3x^3 - 6x^2 + 5x + 6 \) appear to be \(-\frac{3}{2}\), and 2. Substituting these into \( f \) confirms this.

The graph shows that the solution of \( 2x^4 - 3x^3 - 6x^2 + 5x + 6 < 0 \) is \( \left( \frac{3}{2}, 2 \right) \).

\([-5, 5] \text{ by } [-10, 10]\)

20. The zero of \( f(x) = 3x^4 - 5x^3 - 12x^2 + 12x + 16 \) appear to be \(-\frac{4}{3}\), \(-1\), and 2. Substituting these into \( f \) confirms this. The graph shows that the solution of \( 3x^4 - 5x^3 - 12x^2 + 12x + 16 \geq 0 \) is \( (-\infty, -\frac{4}{3}) \cup [-1, \infty) \).

\([-3, 3] \text{ by } [-3, 23]\)

21. \( f(x) = (x^2 + 4)(2x^2 + 3) \)

(a) The solution is \((-\infty, \infty)\), because both factors of \( f(x) \) are always positive.

(b) \((-\infty, \infty)\), for the same reason as in part (a).

(c) There are no solutions, because both factors of \( f(x) \) are always positive.

(d) There are no solutions, for the same reason as in part (c).

22. \( f(x) = (x^2 + 1)(-2 - 3x^2) \)

(a) There are no solutions, because \( x^2 + 1 \) is always positive and \(-2 - 3x^2 \) is always negative.

(b) There are no solutions, for the same reason as in part (a).

(c) \((-\infty, \infty)\), because \( x^2 + 1 \) is always positive and \(-2 - 3x^2 \) is always negative.

(d) \((-\infty, \infty)\), for the same reason as in part (c).

23. \( f(x) = (2x^2 - 2x + 5)(3x - 4)^2 \)

The first factor is always positive because the leading term has a positive coefficient and the discriminant \((-2)^2 - 4(2)(5) = -36\) is negative. The only zero is \( x = 4/3 \), with multiplicity two, since that is the solution for \( 3x - 4 = 0 \).

(a) True for all \( x \neq \frac{4}{3} \)

(b) \((-\infty, \infty)\)

(c) There are no solutions.

Section 2.8 Solving Inequalities in One Variable

24. \( f(x) = (x^2 + 4)(3 - 2x)^2 \)

The first factor is always positive. The only zero is \( x = 3/2 \), with multiplicity two, since that is the solution for \( 3 - 2x = 0 \).

(a) True for all \( x \neq \frac{3}{2} \)

(b) \((-\infty, \infty)\)

(c) There are no solutions.

(d) \( x = \frac{3}{2} \)

25. (a) \( f(x) = 0 \) when \( x = 1 \)

(b) \( f(x) \) is undefined when \( x = -\frac{3}{2}, 4 \)

(c) \( f(x) > 0 \) when \( -\frac{3}{2} < x < 1 \) or \( x > 4 \)

(d) \( f(x) < 0 \) when \( x < -\frac{3}{2} \) or \( 1 < x < 4 \)

\[ \begin{array}{cccccccc}
\text{f(x)} & \text{undefined} & \text{undefined} & \text{undefined} & \text{undefined} \\
\text{Negative} & \text{Positive} & \text{Negative} & \text{Positive} \\
-\frac{3}{2} & 1 & 4 & \frac{3}{2} \\
\end{array} \]

26. (a) \( f(x) = 0 \) when \( x = \frac{7}{2}, -1 \)

(b) \( f(x) \) is undefined when \( x = -5 \)

(c) \( f(x) > 0 \) when \(-5 < x < -1 \) or \( x > \frac{7}{2} \)

(d) \( f(x) < 0 \) when \( x < -5 \) or \(-1 < x < \frac{7}{2} \)

\[ \begin{array}{cccccccc}
\text{f(x)} & \text{undefined} & \text{undefined} & \text{undefined} & \text{undefined} \\
\text{Negative} & \text{Positive} & \text{Negative} & \text{Positive} \\
-5 & -1 & \frac{7}{2} & \frac{3}{2} \\
\end{array} \]

27. (a) \( f(x) = 0 \) when \( x = 0, -3 \)

(b) \( f(x) \) is undefined when \( x < -3 \)

(c) \( f(x) > 0 \) when \( x > 0 \)

(d) \( f(x) < 0 \) when \( -3 < x < 0 \)

\[ \begin{array}{cccccccc}
\text{f(x)} & \text{undefined} & \text{undefined} & \text{undefined} & \text{undefined} \\
\text{Negative} & \text{Positive} & \text{Negative} & \text{Positive} \\
-3 & 0 & \frac{3}{2} & \frac{3}{2} \\
\end{array} \]

28. (a) \( f(x) = 0 \) when \( x = 0, \frac{9}{2} \)

(b) None. \( f(x) \) is never undefined.

(c) \( f(x) > 0 \) when \( x \neq \frac{9}{2}, 0 \)

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120 Chapter 2 Polynomial, Power, and Rational Functions

(d) None. \( f(x) \) is never negative.

\[
\begin{array}{c|ccc}
\text{ } & \text{Positive} & \text{Negative} & \text{Positive} \\
\hline
\frac{-1}{2} & 0 & 0 & 0 \\
0 & + & - & + \\
9 & - & + & - \\
\end{array}
\]

29. (a) \( f(x) = 0 \) when \( x = -5 \)

(b) \( f(x) \) is undefined when \( x = -\frac{1}{2}, x = 1, x < -5 \).

(c) \( f(x) > 0 \) when \(-5 < x < -\frac{1}{2} \) or \( x > 1 \)

(d) \( f(x) < 0 \) when \(-\frac{1}{2} < x < 1 \)

\[
\begin{array}{c|ccc}
\text{ } & \text{Positive} & \text{Negative} & \text{Positive} \\
\hline
-5 & 0 & + & + \\
-\frac{1}{2} & - & + & - \\
1 & - & + & + \\
\end{array}
\]

30. (a) \( f(x) = 0 \) when \( x = 1 \)

(b) \( f(x) \) is undefined when \( x = 4, x = -2 \)

(c) \( f(x) > 0 \) when \(-2 < x < 1 \) or \( x > 4 \)

(d) \( f(x) < 0 \) when \( 1 < x < 4 \)

\[
\begin{array}{c|ccc}
\text{ } & \text{Positive} & \text{Negative} & \text{Positive} \\
\hline
-2 & 0 & - & + \\
-1 & 0 & + & + \\
1 & - & + & - \\
4 & - & + & + \\
\end{array}
\]

31. (a) \( f(x) = 0 \) when \( x = 3 \)

(b) \( f(x) \) is undefined when \( x = 4, x < 3 \)

(c) \( f(x) > 0 \) when \( 3 < x < 4 \) or \( x > 4 \)

(d) None. \( f(x) \) is never negative.

\[
\begin{array}{c|ccc}
\text{ } & \text{Positive} & \text{Negative} & \text{Positive} \\
\hline
3 & 0 & - & 0 \\
4 & + & + & + \\
\end{array}
\]

32. (a) None. \( f(x) \) is never 0

(b) \( f(x) \) is undefined when \( x \leq 5 \).

(c) \( f(x) > 0 \) when \( 5 < x < \infty \)

(d) None. \( f(x) \) is never negative.

\[
\begin{array}{c|cc}
\text{ } & \text{Positive} \\
\hline
5 & + \\
\end{array}
\]

33. \( f(x) = \frac{x - 1}{x^2 - 4} = \frac{x - 1}{(x + 2)(x - 2)} \) has points of potential sign change at \( x = -2, 1, 2 \).

\[
\begin{array}{c|ccc}
\text{ } & \text{Positive} & \text{Negative} & \text{Positive} \\
\hline
-2 & 0 & - & + \\
1 & - & + & + \\
2 & 0 & + & - \\
\end{array}
\]

By the sign chart, the solution of \( \frac{x - 1}{x^2 - 4} < 0 \) is \((\infty, -2) \cup (1, 2)\).

34. \( f(x) = \frac{x + 2}{x^2 - 9} = \frac{x + 2}{(x + 3)(x - 3)} \) has points of potential sign change at \( x = -3, -2, 3 \).

\[
\begin{array}{c|ccc}
\text{ } & \text{Positive} & \text{Negative} & \text{Positive} \\
\hline
-3 & 0 & - & + \\
-2 & 0 & + & + \\
3 & + & + & - \\
\end{array}
\]

By the sign chart, the solution of \( \frac{x + 2}{x^2 - 9} < 0 \) is \((\infty, -3) \cup (-2, 3)\).

35. \( f(x) = \frac{x^2 - 1}{x^2 + 1} = \frac{(x + 1)(x - 1)}{(x^2 + 1)} \) has points of potential sign change at \( x = -1, 1 \).

\[
\begin{array}{c|ccc}
\text{ } & \text{Positive} & \text{Negative} & \text{Positive} \\
\hline
-1 & 0 & - & 0 \\
1 & 0 & + & 0 \\
\end{array}
\]

By the sign chart, the solution of \( \frac{x^2 - 1}{x^2 + 1} \leq 0 \) is \([-1, 1]\).

36. \( f(x) = \frac{x^2 - 4}{x^2 + 4} = \frac{(x + 2)(x - 2)}{x^2 + 4} \) has points of potential sign change at \( x = -2, 2 \).

\[
\begin{array}{c|ccc}
\text{ } & \text{Positive} & \text{Negative} & \text{Positive} \\
\hline
-2 & 0 & - & 0 \\
2 & 0 & + & 0 \\
\end{array}
\]

By the sign chart, the solution of \( \frac{x^2 - 4}{x^2 + 4} > 0 \) is \((\infty, -2) \cup (2, \infty)\).

37. \( f(x) = \frac{x^2 + x - 12}{x^2 - 4x + 4} = \frac{(x + 4)(x - 3)}{(x - 2)^2} \) has points of potential sign change at \( x = -4, 2, 3 \).

\[
\begin{array}{c|ccc}
\text{ } & \text{Positive} & \text{Negative} & \text{Positive} \\
\hline
-4 & 0 & - & 0 \\
2 & - & 0 & + \\
3 & 0 & + & - \\
\end{array}
\]

By the sign chart, the solution of \( \frac{x^2 + x - 12}{x^2 - 4x + 4} > 0 \) is \((\infty, -4) \cup (3, \infty)\).
38. \( f(x) = \frac{x^2 + 3x - 10}{x^2 - 6x + 9} = \frac{(x + 5)(x - 2)}{(x - 3)^2} \) has points of potential sign change at \( x = -5, 2, 3. \)

39. \( f(x) = \frac{x^3 - x}{x^2 + 1} = \frac{x(x + 1)(x - 1)}{x^2 + 1} \) has points of potential sign change at \( x = -1, 0, 1. \)

40. \( f(x) = \frac{x^3 - 4x}{x^2 + 2} = \frac{x(x + 2)(x - 2)}{x^2 + 2} \) has points of potential sign change at \( x = -2, 0, 2. \)

41. \( f(x) = |x - 2| \) has points of potential sign change at \( x = 0, 2. \)

42. \( f(x) = \frac{x - 3}{|x + 2|} \) has points of potential sign change at \( x = -2, 3. \)

43. \( f(x) = (2x - 1)\sqrt{x + 4} \) has a point of potential sign change at \( x = \frac{1}{2}. \) Note that the domain of \( f \) is \([-4, \infty). \)

44. \( f(x) = (3x - 4)\sqrt{2x + 1} \) has a point of potential sign change at \( x = \frac{4}{3}. \) Note that the domain of \( f \) is \( \left[ -\frac{1}{2}, \infty \right). \)

45. \( f(x) = \frac{x^5(x - 2)}{(x + 3)^2} \) has points of potential sign change at \( x = -3, 0, 2. \)

46. \( f(x) = \frac{(x - 5)^4}{x(x + 3)} \) has points of potential sign change at \( x = -3, 0, 5. \)

47. \( f(x) = x^2 - \frac{2}{x} = \frac{x^3 - 2}{x} \) has points of potential sign change at \( x = 0, \sqrt[3]{2}. \)
122 Chapter 2 Polynomial, Power, and Rational Functions

By the sign chart, the solution of \( x^2 - \frac{2}{x} > 0 \) is 
\(-\infty, 0) \cup (\sqrt{2}, \infty)\).

48. \( f(x) = x^2 + \frac{4}{x} = \frac{x^3 + 4}{x} \) has points of potential sign change at \( x = 0, -\sqrt{4} \).

\[
\begin{array}{c|c|c|c|c}
\text{Sign Chart} & (\cdot) & 0 & (\oplus) & (\otimes) \\
\hline
\text{Positive} & -\sqrt{4} & 0 & \text{Positive} & \text{Positive} \\
\end{array}
\]

49. \( f(x) = \frac{1}{x + 1} + \frac{1}{x - 3} = \frac{2(x - 1)}{(x + 1)(x - 3)} \) has points of potential sign change at \( x = -1, 1, 3 \).

\[
\begin{array}{c|c|c|c|c}
\text{Sign Chart} & (\cdot) & 0 & (\oplus) & (\otimes) \\
\hline
\text{Negative} & x & 3 & \text{Positive} & \text{Positive} \\
\end{array}
\]

50. \( f(x) = \frac{1}{x + 2} - \frac{2}{x - 1} = \frac{-x - 5}{(x + 2)(x - 1)} \) has points of potential sign change at \( x = -5, -2, 1 \).

\[
\begin{array}{c|c|c|c|c}
\text{Sign Chart} & (\cdot) & 0 & (\otimes) & (\oplus) \\
\hline
\text{Positive} & 5 & -2 & 1 & \text{Negative} \\
\end{array}
\]

51. \( f(x) = (x + 3)|x - 1| \) has points of potential sign change at \( x = 3, -1 \).

\[
\begin{array}{c|c|c|c|c}
\text{Sign Chart} & (\otimes) & 1 & 0 & (\oplus) \\
\hline
\text{Positive} & 3 & \text{Positive} & 1 & \text{Positive} \\
\end{array}
\]

52. \( f(x) = (3x + 5)^2|x - 2| \) is always 0 or positive since \((3x + 5)^2 \geq 0 \) for all real \( x \) and \(|x - 2| \geq 0 \) for all real \( x \). Thus the inequality \((3x + 5)^2|x - 2| < 0 \) has no solution.

53. \( f(x) = \frac{(x - 5)(x - 2)}{\sqrt{2x - 2}} \) has points of potential sign change at \( x = 2, 5 \). Note that the domain of \( f \) is \((1, \infty)\).

\[
\begin{array}{c|c|c|c|c|c}
\text{Sign Chart} & (\cdot) & 0 & (\otimes) & (\oplus) & (\otimes) \\
\hline
\text{Undefined} & -1 & 2 & 5 & \text{Negative} & \text{Negative} \\
\end{array}
\]

54. \( f(x) = \frac{x^2(x - 4)^3}{\sqrt{x + 1}} \) has points of potential sign change at \( x = 0, 4 \). Note that the domain of \( f \) is \((-\infty, \infty)\).

\[
\begin{array}{c|c|c|c|c|c}
\text{Sign Chart} & (\cdot) & 0 & (\otimes) & (\oplus) & (\oplus) \\
\hline
\text{Undefined} & -1 & 0 & 4 & \text{Negative} & \text{Positive} \\
\end{array}
\]

55. One way to solve the inequality is to graph \( y = 3(x - 1) + 2 \) and \( y = 5x + 6 \) together, then find the interval along the \( x \)-axis where the first graph is below or intersects the second graph. Another way is to solve for \( x \) algebraically.

56. Let \( x \) be the number of hours worked. The repair charge is \( 25 + 18x \); this must be less than $100. Starting with \( 25 + 18x < 100 \), we have \( 18x < 75 \), so \( x < 4.166 \). Therefore the electrician worked no more than 4 hours 7.5 minutes (which rounds to 4 hours).

57. Let \( x > 0 \) be the width of a rectangle; then the length is \( 2x - 2 \) and the perimeter is \( P = 2[x + (2x - 2)] \). Solving \( P < 200 \) and \( 2x - 2 > 0 \) (below) gives \( 1 < x < 30 \) in.

58. Let \( x \) be the number of candy bars made. Then the costs are \( C = 0.13x + 2000 \), and the income is \( I = 0.35x \). Solving \( C < I \) (below) gives \( x > 9090.91 \). The company will need to make and sell 9091 candy bars to make a profit.

59. The lengths of the sides of the box are \( x, 12 - 2x, \) and \( 15 - 2x \), so the volume is \( x(12 - 2x)(15 - 2x) \). To solve \( x(12 - 2x)(15 - 2x) \leq 100 \), graph \( f(x) = x(12 - 2x)(15 - 2x) - 100 \) and find the zeros: \( x \approx 0.69 \) and \( x \approx 4.20 \).
Section 2.8  Solving Inequalities in One Variable  123

From the graph, the solution of \( f(x) \leq 0 \) is approximately \([0, 0.69) \cup [4.20, 6]\). The squares should be such that either 0 in \( x \leq 0.69 \) in or 4.20 in \( x \leq 6 \) in.

60. The circumference of the base of the cone is \( 8\pi - x \),

\[
r = 4 - \frac{x}{2\pi}, \quad h = \sqrt{16 - \left(4 - \frac{x}{2\pi}\right)^2}.
\]
The volume is

\[
V = \pi x r^2 h = \pi x \left(4 - \frac{x}{2\pi}\right) \sqrt{16 - \left(4 - \frac{x}{2\pi}\right)^2}.
\]

To solve \( v \geq 21 \), graph \( v - 21 \) and find the zeros: \( x \approx 1.68 \) in. or \( x \approx 9.10 \).

From the graph, the solution of \( v - 21 \geq 0 \) is approximately \([1.68, 9.10]\). The arc length should be in the range of \( 1.68 \) in. \( x 9.10 \) in.

61. (a) \( L = 500 \) cm

\[
V = \pi x^2 h = 500 \Rightarrow h = \frac{500}{\pi x^2}.
\]

\[
S = 2\pi x h + 2\pi x^2 = 2\pi x \left(\frac{500}{\pi x^2}\right) + 2\pi x^2 = \frac{1000}{x} + 2\pi x^2.
\]

(b) Solve \( S < 900 \) by graphing \( \frac{1000}{x} + 2\pi x^2 \) - 900 and finding its zeros:

\[
x \approx 1.12 \text{ and } x \approx 11.37
\]

From the graph, the solution of \( S < 900 \) is approximately \((1.12, 11.37)\). So the radius is between 1.12 cm and 11.37 cm. The corresponding height must be between 1.23 cm and 126.88 cm.

(c) Graph \( S \) and find the minimum graphically.

62. (a) \( \frac{1}{R} = \frac{1}{2.3} + \frac{1}{x} \)

\[2.3x = R(x + 2.3) \]

\[R = \frac{2.3x}{x + 2.3} \]

(b) \( R \geq 1.7 \Rightarrow \frac{2.3x}{x + 2.3} \geq 1.7 \)

\[\frac{2.3x}{x + 2.3} \geq 1.7 \]

\[\frac{2.3x}{x + 2.3} - 1.7 \geq 0 \]

\[\frac{2.3x - 1.7(x + 2.3)}{x + 2.3} \geq 0 \]

The function \( f(x) = \frac{0.6x - 3.91}{x + 2.3} \) has a point of potential sign change at \( x = \frac{391}{60} \approx 6.5 \). Note that the domain of \( f \) is \((-2.3, \infty)\).

\[
\begin{array}{c|c|c|c}
\text{Undefined} & (-) & 0 & (+) \\
\text{Negative} & (--) & 0 & (+)
\end{array}
\]

By the sign chart, the solution of \( f(x) \leq 0 \) is about \([6.5, \infty)\). The resistance in the second resistor is at least 6.5 ohms.

63. (a) \( y = 2.726x + 282.132 \)

\[
(0, 9) \text{ by } (270, 320)
\]

(b) From the graph of \( y = 2.726x + 282.132 \), we find that \( y = 315 \) when \( x \approx 12.057 \), which is approximately 12 years and 21 days. The population will exceed 315 million on about July 22, 2012.

64. (a) \( y = -5053.57x^2 + 63,355.00x^2 + 49,724.28 \)

\[
(2, 9) \text{ by } (100000, 300000)
\]
124 Chapter 2 Polynomial, Power, and Rational Functions

(b) From the graph of \( y = -5053.57x^2 + 63,355.00x + 49,724.28 \), we find that \( y = 200,000 \) when \( x \approx 9.36 \).

The median cost of a new home will return to $200,000 in late 2009.

65. True. Because the factor \( x^4 \) has an even power, it does not change sign at \( x = 0 \).

67. \( x \) must be positive but less than 1. The answer is C.

68. The statement is true so long as the denominator does not equal zero. The answer is B.

69. The statement is true so long as the denominator is negative and the numerator is nonzero. Thus \( x \) must be less than three but nonzero. The answer is D.

70. The expression \((x^2 - 1)^2\) cannot be negative for any real \( x \), and it can equal zero only for \( x = \pm 1 \). The answer is A.

71. \( f(x) = \frac{(x - 1)(x + 2)^2}{(x - 3)(x + 1)} \)

Vertical asymptotes: \( x = -1, x = 3 \)

x-intercepts: \((-2, 0), (1, 0)\)

y-intercept: \((0, \frac{4}{3})\)

\[ \begin{array}{c|c|c|c|c}
\text{Sign of } x & \text{Sign of } (x-3) & \text{Sign of } (x+1) & \text{Sign of } (x-1) & \text{Sign of } (x+2) \\
\hline
\text{Positive} & \text{Negative} & \text{Positive} & \text{Negative} & \text{Positive} \\
\end{array} \]

By hand:

Grapher:

72. \( g(x) = \frac{(x - 3)^3}{x^3 + 4x} = \frac{(x - 3)^3}{x(x + 4)} \)

Vertical asymptotes: \( x = -4, x = 0 \)

x-intercept: \((3, 0)\)

y-intercept: None

73. (a) \( |x - 3| < 1/3 \Rightarrow |3x - 9| < 1 \Rightarrow |f(x) - 4| < 1 \).

For example:

\( |f(x) - 4| = |(3x - 5) - 4| = |3x - 9| \)

\( 3|x - 3| < 3 \left( \frac{1}{3} \right) = 1 \)

(b) If \( x \) stays within the dashed vertical lines, \( f(x) \) will stay within the dashed horizontal lines. For the example in part (a), the graph shows that for:

\( \frac{8}{3} < x < \frac{10}{3} \) (that is, \( |x - 3| < \frac{1}{3} \)), we have

\( 3 < f(x) < 5 \) (that is, \( |f(x) - 4| < 1 \)).

(c) \( |x - 3| < 0.01 \Rightarrow |3x - 9| < 0.03 \Rightarrow |f(x) - 4| < 0.03 \). The dashed lines would be closer to \( x = 3 \) and \( y = 4 \).

75. One possible answer: Given \( 0 < a < b \), multiplying both sides of \( a < b \) by \( a \) gives \( a^2 < ab \); multiplying by \( b \) gives \( ab < b^2 \). Then, by the transitive property of inequality, we have \( a^2 < b^2 \).

76. One possible answer: Given \( 0 < a < b \), multiplying both sides of \( a < b \) by \( \frac{1}{ab} \) gives \( \frac{a}{b} < \frac{1}{a} \), which is equivalent to \( \frac{1}{a} > \frac{1}{b} \).